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DETERMINATION OF SOLID-PHASE CONCENTRATION DISTRIBUTION IN A MOVING SUSPENSION

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Certain properties of the equations of motion and the boundary conditions for a slowly flowing suspension are examined with allowance for sedimentation. The method obtained is applied to the calculation of the sedimentation boundary layer near an inclined plate.

In spite of the multitude of applications (chemical engineering, water-purification, hydrology), methods of calculating concentrations in moving suspensions have not as yet been sufficiently developed. Below, certain simple laws relating to the slow, nonturbulent flow of suspensions are investigated.

General equations of motion and boundary conditions. The motion of a suspension differs from that of a homogenous fluid only in that the suspended particles may move relative to the surrounding liquid under the influence of some force field (inertial, gravitational, centrifugal). In relation to the slow laminar mixing examined here, inertia forces are usually negligible. Thus it is necessary to add two equations to the general equations of hydrodynamics* in this case: the equation of relative motion of the suspended particles in the potential force field

$$\vec{w} = \vec{v} + \vec{w}_0 \tag{1}$$

and the continuity equation for the suspended particles

$$\frac{\partial c}{\partial t} = \operatorname{div}(c \cdot \vec{w}). \tag{2}$$

The continuity equation for the liquid is modified somewhat owing to the suspension:

$$\operatorname{div}[(1 - c) \cdot \vec{v}] = 0; \tag{3a}$$

at small concentrations we can assume

$$\operatorname{div} \vec{v} \approx 0. \tag{3b}$$

In contrast to the usual situation, the density of the suspension, which is a direct function of the mass concentration of suspended particles, may vary considerably from point to point. Buoyancy forces are therefore important. We shall evaluate the order of magnitude of the viscous and buoyancy forces (per unit volume):

$$f_i = \mu \sum_{j \neq i} \frac{\partial^2 v_i}{\partial x_j^2} \sim \mu \frac{V}{l^2}, \tag{4}$$

$$f_{\text{b}} = g \Delta \rho. \tag{5}$$

Equating (4) and (5), we obtain the order of magnitude of the convective mixing rate

$$v_{\text{c}} \sim l^2 g \Delta\rho/\mu. \tag{6}$$

Equation (6) shows that remote from the walls of the container, the characteristic convective mixing rate may attain large values even at small values of $\Delta\rho$. The convective mixing then acts so as to equalize the concentration in planes perpendicular to the vector $\vec{g}$, i.e., only the concentration distribution for which

$$[\vec{g} \times \operatorname{grad} c] = 0 \tag{7**}$$

is stable. Moreover, it is clear that the state where the vectors $\vec{g}$ and grad c are parallel, but opposed is also unstable.

Another situation obtains near the walls of the container (small values of l). Here the convective mixing rate is limited by the considerable viscous friction at the wall. Therefore, near the container walls considerable solid-phase

* With averaged physical parameters for the suspension.

** The validity of (7) may also be found directly from an examination of the circulation forces around the edge of the convective flow.

concentration inhomogeneities may be created by disturbing factors (and it will become clear from the following that certain parts of the container walls constitute such factors).

In the suspension case the usual boundary conditions applicable to the motion of a homogeneous fluid retain their force for the dispersion medium, but not for the disperse phase. Indeed, it is obvious that the normal and tangential components of the suspended particle velocity cannot vanish at the fixed or moving boundary (i.e., the phase interface). It can be seen from (2) that at the fixed boundary ($\vec{v}_s = 0$)

$$\vec{w}_s = \vec{w}_0, \tag{8}$$

and at the moving boundary ($\vec{v}_{s.p} = 0$):

$$\vec{w}_{s.p} = \vec{w}_{0p}, \tag{9}$$

where the subscript "p" denotes projection of the vector on the normal to the boundary surface. Physically, this means that on surface elements having a projection in the direction $\vec{\omega}_0$, there is formed either a sediment [$(\vec{\omega}_0 \cdot n) > 0$, "bottom" surfaces], or layers of pure dispersion medium [$(\vec{\omega}_0, \cdot \vec{n}) < 0$, "top" surfaces]. A "bottom" surface is thus equivalent to a surface of suspension sinks, the surface sink intensity being (for the solid phase and the dispersion medium)

$$N_{sp}^{-} = c(\vec{w}_0 \cdot \vec{n})^{**}, \tag{10}^*$$

$$N^{-} = (1 - c)(\vec{w}_0 \cdot \vec{n}). \tag{11}$$

Here c denotes the solid-phase concentration near the surface. The "top" surface is equivalent to a surface of sources of pure dispersion medium with surface intensity:

$$N^{+} = -(\vec{w}_0 \cdot \vec{n})^{*}. \tag{12}^{**}$$

In general, by representing the boundary surfaces [or, more exactly, their elements, where $(\vec{\omega}_0 \cdot \vec{n}) \neq 0$], as surfaces of fictitious sources and sinks, it is possible, at least near the surfaces, to reduce the equations of motion of the solid phase in a suspension to the equations of motion of a dissolved substance in a two-component liquid (except for diffusion).

In the equations of motion of a suspension it is essential to take into account relative motion of the solid phase in the dispersion medium (velocity $\vec{\omega}_0$), but the introduction of these fictitious sources and sinks is equivalent to allowing for relative motion. Indeed, by introducing fictitious sources and sinks of appropriate intensity [equations (10) and (12)], and assuming no relative motion between phases (for example, the case of equal densities of both phases), we obtain the very same boundary conditions and the same equations for the concentration field as for the real suspension with relative phase velocity ω_0 [Eqs. (8) and (9)].

A detailed analysis shows that the method of introducing fictitious sources on the surfaces bounding the suspension, as proposed above, is also applicable in the case of a hydrodynamic boundary layer, i.e., when the use of (8) is not quite accurate.

It is appropriate to explain the physical meaning of introducing sources and sinks on the bounding surfaces. There is an actual flow of pure dispersion medium through the interface between the medium and the suspension due to deposition of solid particles. This flow is replaced by an equal flow of fictitious sources, and at the same time it is assumed that the velocity $\vec{\omega}_0$ is zero; a certain lack of coincidence between the surfaces on which the actual and fictitious sources act is not important.

The method of fictitious sources may also be applied to polydisperse suspensions. In this case, however, it is necessary to introduce fictitious sources not only of pure dispersion medium, but also of suspension, with concentration different from that of the suspension near the boundary surfaces. In this case, the fictitious sources are distributed throughout the sedimentation boundary layer.

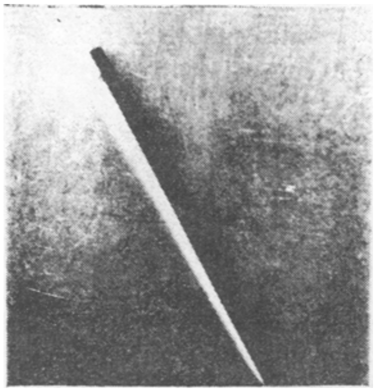
The above-mentioned idea of the role of boundary surfaces as sources of solid-phase material is a very fruitful one in a number of concrete problems concerning laminar motion of a suspension, as it permits the desired solution to be obtained rapidly (for example, the problem of thermal convection in the interior of the suspension).

The sedimentation boundary layer. As mentioned above, convective flows very strongly equalize nonuniformities of concentration of a suspension due to various causes (in addition to the case when grad c is parallel to the vector $\vec{g}$). Appreciable concentration nonuniformities (appreciable compared to the concentration itself) may exist in a quasi-

* The case of large solid-phase concentrations, where conditions of restricted deposition may apply, is not considered.

** A similar interpretation was given in [1] without derivation and in less general form.

steady state only near the bounding surfaces and under the action of disturbing factors. The presence in the volume of the suspension of surfaces where $(\vec{g} \cdot \vec{n}) \neq 0$ is a constant disturbing factor of this type.



An inclined plate in a vessel with a settling suspension (the light layer under the plate is the sedimentation boundary layer).

Let us examine the problem of a sedimentation boundary layer near an inclined "top" plane in a monodisperse suspension of density exceeding that of the dispersion medium (see figure).

It follows from the account given above that a pure layer of dispersion medium is formed near "top" surfaces. Volume buoyancy forces acting in the layer of dispersion medium create a convective flow whose thickness is small compared to the length of the inclined surface. This enables one to simplify the Navier-Stokes equation considerably.

In addition, the problem reduces to two dimensions if one of the rectangular coordinate axes, z, is directed along a horizontal line (in general perpendicular to vector $\vec{g}$) lying in the inclined plane. The x axis lies in the inclined plane and is taken to be the direction of flow of the liquid in the sedimentation boundary layer, i.e., upwards along the inclined plane, while the y axis is perpendicular to it and is taken downwards from the inclined plane. Thus, the equation of the sedimentation boundary layer near the inclined plate (inclined at angle α to the horizontal) may be written as

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2} + g \frac{\Delta\rho}{\rho} \sin\alpha. \tag{13}$$

The continuity equation for the dispersion medium (small concentration of suspension, $\Delta\rho \ll \rho$) is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0. \tag{14}$$

The boundary conditions at the solid wall are obvious (v_x, $v_y = 0$), and at the outer edge of the boundary layer it is assumed that

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=y_\text{r}} = 0. \tag{15}$$

The location of the outer edge is determined from the total intensity of the fictitious sources over the whole inclined surface:

$$\int_0^{y_s} v_x dy = w_0 x \cos\alpha. \tag{16}$$

On the left is the fluid flux in the sedimentation boundary layer for some value of the coordinate $x = x_1$, and on the right the fictitious source flux for the section of the inclined plane from $x = 0$ to $x = x_1$. These fluxes are equal, since the flux in the layer is formed precisely by the action of the sources on the whole of the underlying part of the inclined plane. Here the fluxes are expressed in two-dimensional form, since the flow does not depend on the coordinate z, i.e., we write simply dy instead of the element of cross-sectional area of the layer $ds = z \cdot dy$, and x instead of the area of the inclined surface $F = z \cdot x$.

Let us determine when the inertia terms in (13) can be neglected. The first two terms are of order v_x^2/l, and the third $\nu v_x/\delta^2$, where l and δ are, respectively, the length and thickness of the boundary layer. The ratio of the inertial to the viscous terms is

$$\frac{v_x \partial v_x/\partial x}{\nu\, \partial^2 v_x/\partial y^2} \sim \frac{v_x \delta^2}{\nu l} \sim \frac{w_0 \delta \cos\alpha}{\nu} \equiv S, \tag{17}$$

since $w_0 l \cos\alpha \sim v_x \delta$.

For values of the boundary layer sedimentation parameter $S \ll 1$ the inertia terms may be omitted:

$$\nu \frac{\partial^2 v_x}{\partial y^2} + \frac{g\Delta\rho}{\rho} \sin\alpha = 0. \tag{18}$$

The solution of (18) has the form

$$v_x = \frac{g\,\Delta\rho \sin\alpha}{\mu}\left(y_r - \frac{y}{2}\right) y, \tag{19}$$

$$y_s = \sqrt[3]{\frac{3w_0 x\,\mu \operatorname{ctg}\alpha}{\Delta\rho\, g}}\,. \tag{20}$$

As far as (13) is concerned, the approximate methods developed in hydrodynamic boundary layer theory may be applied.

The results obtained can be used in various branches of chemical engineering, where a solid phase is separated from a liquid by setting methods, in a gravity or centrifugal force field (centrifugal separator). Because baffles create sedimentation boundary layers, their introduction increases the effectiveness of settling, as practice bears out [1, 3]. A knowledge of the sedimentation boundary layer is required to make a correct choice of slope angle and number of baffles in centrifuges and settling tanks. The sedimentation boundary layer may also be used to avoid formation of dense deposits on the hot surfaces of heat exchangers designed for suspensions.

NOTATION

l—characteristic distance from wall of container; V—characteristic velocity; g—acceleration due to gravity; $\Delta\rho$—difference between density of liquid in boundary layer and average density (averaged over plane perpendicular to vector $\vec{g}$), in particular, difference between density of suspension and that of dispersion fluid; y_s—thickness of boundary layer (layer of pure dispersion medium) at the "top" of the inclined plane; ν and μ—kinematic and dynamic viscosity of dispersion medium, respectively; α—angle of inclination of surface (from the horizontal); $\vec{\omega}$—absolute velocity of solid-phase particles; $\vec{\omega}_0$—velocity of relative motion of solid-phase particles and of liquid (sedimentation rate); $\vec{v}$—absolute velocity of liquid; c—volume concentration of solid; $\vec{f}$—volume density of viscous force vector; $\vec{f}_b$—volume density of buoyancy force vector; ρ—density; t—time; N^-—surface strength of sinks; N^+—surface strength of sources. Subscripts: i, j, k or x, y, z—vector components in rectangular coordinates; s—phase interface; p—projections of vectors on normal to surface, outwards with respect to liquid volume.

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